Reversibilization in Functional and Concurrent Programming

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A collaborative work...

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COST action IC1405 on Reversible Computation

Reversible programming languages

Each execution step is **reversible**

Backward steps must be **deterministic**

E.g., **Janus**: `if` c_1 `then` s_1 `else` s_2 `fi` c_2

Reversible languages are not universal
(e.g., cannot compute non-injective functions)

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Reversibilization

Given an irreversible programming language **L** with semantics **Sem** over states $s_0, s_1, \ldots, s_n \in \textbf{State}$:

$$s_0 \to s_1 \to \ldots \to s_n$$

we can extend the states with enough information so that $\textbf{Sem}^R$ over $\langle s_0, h_0 \rangle, \langle s_1, h_1 \rangle, \ldots, \langle s_n, h_n \rangle \in \textbf{State}'$:

$$\langle s_0, [\,] \rangle \to \langle s_1, [s_0] \rangle \to \ldots \to \langle s_n, [s_{n-1}, \ldots, s_0] \rangle$$

becomes reversible

This is known as a Landauer embedding and is the main technique for **reversibilization**

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It may seem impractical at first. . .

However,

- in some cases, performance is not critical (e.g., debugging)
- in some other cases, the history can be optimized (e.g., store nothing when applying an injective function)
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Functional Programming

A first-order, eager functional language

Defining a Landauer embedding

Functions defined by pattern-matching, e.g.,

$$
\begin{array}{rcl}
\mathit{add}(0, y) & \to & y \\
\mathit{add}(\mathrm{s}(x), y) & \to & \mathrm{s}(\mathit{add}(x, y)) \\
\mathit{fst}(x, y) & \to & x
\end{array}
$$

An example reduction:

$$\mathit{fst}(\underline{\mathit{add}(\mathrm{s}(0), 0)}, 0) \to \mathit{fst}(\mathrm{s}(\underline{\mathit{add}(0, 0)}), 0) \to \underline{\mathit{fst}(\mathrm{s}(0), 0)} \to \mathrm{s}(0)$$

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An example reduction:

$$\boxed{fst(add(s(0), 0), 0) \leftarrow fst(s(add(0, 0)), 0)} \rightarrow fst(s(0), 0) \rightarrow s(0)$$

What should include a Landauer embedding?

Defining a Landauer embedding

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What should include a Landauer embedding?

$\Rightarrow$ position of reduced expression

Defining a Landauer embedding

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$$\begin{array}{rrcl} \beta_1 : & add(0, y) & \leftarrow & y \\ \beta_2 : & add(s(x), y) & \leftarrow & s(add(x, y)) \\ \beta_3 : & fst(x, y) & \leftarrow & x \end{array}$$

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$$

An example reduction:

$$
\mathit{fst}(\mathit{add}(\mathsf{s}(0), 0), 0) \leftarrow \mathit{fst}(\mathsf{s}(\mathit{add}(0, 0)), 0) \leftarrow \boxed{\mathit{fst}(\mathsf{s}(0), 0) \leftarrow \mathsf{s}(0)}
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What should include a Landauer embedding?

$\Rightarrow$ position of reduced expression, rule, erased values

We store a trace term $\beta(p,\sigma)$ at every reduction step:

$$
\begin{array}{l}
\langle \mathit{fst}(\mathit{add}(\mathsf{s}(0),0),0),[\,]\rangle \\
\qquad \rightharpoonup \langle \mathit{fst}(\mathsf{s}(\mathit{add}(0,0)),0),[\beta_2(1,\mathit{id})]\rangle \\
\qquad \rightharpoonup \langle \mathit{fst}(\mathsf{s}(0),0),[\beta_1(1.1,\mathit{id}),\beta_2(1,\mathit{id})]\rangle \\
\qquad \rightharpoonup \langle \mathsf{s}(0),[\beta_3(\epsilon,\{y\mapsto 0\}),\beta_1(1.1,\mathit{id}),\beta_2(1,\mathit{id})]\rangle
\end{array}
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where

- $\rightharpoonup$ is the reversible *forward* reduction relation
- $\leftharpoondown$ is the reversible *backward* reduction relation

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Can we move from the instrumented semantics to an instrumented program?

I.e., given a program $\mathcal{R}$, define $\mathcal{R}_f$ and $\mathcal{R}_b$ such that

$$\langle s_1, \pi_1 \rangle \rightharpoonup_{\mathcal{R}} \langle s_2, \pi_2 \rangle \quad \text{iff} \quad \langle s_1, \pi_1 \rangle \rightarrow_{\mathcal{R}_f} \langle s_2, \pi_2 \rangle$$

and

$$\langle s_2, \pi_2 \rangle \leftharpoondown_{\mathcal{R}} \langle s_1, \pi_1 \rangle \quad \text{iff} \quad \langle s_2, \pi_2 \rangle \rightarrow_{\mathcal{R}_b} \langle s_1, \pi_1 \rangle$$

Instrumenting the rules to store the applied rule and the erased values is easy (static)

…but storing positions is rather difficult (dynamic)

Alternative: program transformation (flattening), e.g.,

$$\begin{array}{rcl} add(0,y) & \to & y \\ add(s(x),y) & \to & s(add(x,y)) \end{array}$$

$$\Downarrow$$

$$\begin{array}{rcl} add(0,y) & \to & y \\ add(s(x),y) & \to & s(z) \Leftarrow add(x,y) \twoheadrightarrow z \end{array}$$

so that all function calls occur at root positions

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Thus we can get rid of positions in trace terms...

$$\beta(p, \sigma) \;\Rightarrow\; \beta(\sigma)$$

Conditional Programs

A conditional rule:

$$f(s_0) \to r \Leftarrow f_1(s_1) \twoheadrightarrow t_1, \ldots, f_n(s_n) \twoheadrightarrow t_n$$

is equivalent to (Haskell-like):

$$f\ s_0 = r \textbf{ where } t_1 = f_1\ s_1, \ldots, t_n = f_n\ s_n$$

or

$$f\ s_0 = \textbf{let } t_1 = f_1\ s_1, \ldots, t_n = f_n\ s_n \textbf{ in } r$$

E.g.,

$$\begin{array}{rcl} add\ 0\ y & = & y \\ add\ (\mathsf{s}\ x)\ y & = & \textbf{let } z = add\ x\ y \textbf{ in } \mathsf{s}(z) \end{array}$$

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Injectivization & Inversion

Injectivization

We replace each rule

$$\beta : \mathsf{f}(s_0) \to r \Leftarrow \mathsf{f}_1(s_1) \twoheadrightarrow t_1, \ldots, \mathsf{f}_\mathsf{n}(s_n) \twoheadrightarrow t_n$$

by a new rule of the form

$$\mathsf{f}^i(s_0) \to \langle r, \beta(\overline{y}, \overline{w_n})\rangle \Leftarrow \mathsf{f}^i_1(s_1) \twoheadrightarrow \langle t_1, w_1\rangle, \ldots, \mathsf{f}^i_\mathsf{n}(s_n) \twoheadrightarrow \langle t_n, w_n\rangle$$

where $\{\overline{y}\} = (\mathcal{V}ar(s_0) \backslash \mathcal{V}ar(r, s_n, t_n)) \cup \bigcup_{i=1}^{n} \mathcal{V}ar(t_i) \backslash \mathcal{V}ar(r, s_{i+1,n})$

Inversion

We replace each rule

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by a new rule of the form

$$\mathsf{f}^{-1}(r, \beta(\overline{y}, \overline{w_n})) \to \langle s_0\rangle \Leftarrow \mathsf{f}_\mathsf{n}^{-1}(t_n, w_n) \twoheadrightarrow \langle s_n\rangle, \ldots, \mathsf{f}_1^{-1}(t_1, w_1) \twoheadrightarrow \langle s_1\rangle$$

Injectivization & Inversion: An example

$$
\begin{array}{rrcl}
\beta_1: & \mathrm{add}(0,y) & \to & y \\
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More details in "Nishida, Palacios & Vidal: Reversible computation in term rewriting. *J. Log. Algebr. Meth. Program.* 94: 128-149 (2018)"

Application: bidirectionalization

Bidirectional transformations (Bx)

We have two data representations, called source and view with source $\supseteq$ view (assymmetric case)

Function get : source $\mapsto$ view

Consistency: $s \in$ source is consistent with $v \in$ view if $get(s) = v$

We accept updates in both the source and the view $\Rightarrow$ recover consistency!

Function put : view $\times$ source $\mapsto$ source

$$\begin{tikzcd} s \arrow[r, "get"] \arrow[d, "update"'] & v \\ s' \arrow[r, "get"] & v' \end{tikzcd}$$

$$\begin{tikzcd} s \arrow[r, "get"] \arrow[d, bend left] & v \arrow[d, "update"] \\ s' & v' \arrow[l, "put"] \end{tikzcd}$$

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Motivation

Defining the right "put" is not easy

$\Rightarrow$ (syntactic) **bidirectionalization**

Stepwise approach to bidirectionalization

Example (first names)

$$\begin{array}{l} \mathrm{fn}([\,]) \to [\,] \\ \mathrm{fn}(\mathrm{person}(n, l) : xs) \to n : ys \Leftarrow \mathrm{fn}(xs) \twoheadrightarrow ys \\ \mathrm{fn}(\mathrm{city}(c) : xs) \to ys \Leftarrow \mathrm{fn}(xs) \twoheadrightarrow ys \end{array}$$

E.g., given
$s = [\mathrm{person}(\mathrm{john}, \mathrm{smith}), \mathrm{city}(\mathrm{london}), \mathrm{person}(\mathrm{ada}, \mathrm{lovelace})]$,
we have $\mathrm{fn}(s) = [\mathrm{john}, \mathrm{ada}]$

Stepwise approach to bidirectionalization

Example (first names, injective version)

$$\mathsf{fn}^{\mathsf{i}}([\,]) \to \langle [\,], \beta_1 \rangle$$
$$\mathsf{fn}^{\mathsf{i}}(\mathsf{person}(n, l) : xs) \to \langle n : ys, \beta_2(l, w) \rangle \Leftarrow \mathsf{fn}^{\mathsf{i}}(xs) \twoheadrightarrow \langle ys, w \rangle$$
$$\mathsf{fn}^{\mathsf{i}}(\mathsf{city}(c) : xs) \to \langle ys, \beta_3(c, w) \rangle \Leftarrow \mathsf{fn}^{\mathsf{i}}(xs) \twoheadrightarrow \langle ys, w \rangle$$

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(according to [Bancilhon & Spyratos, 1981])

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$$
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\mathsf{fn}^{\mathsf{i}}([\,]) \to \langle [\,], \beta_1 \rangle \\
\mathsf{fn}^{\mathsf{i}}(\mathsf{person}(n,l):xs) \to \langle n:ys, \beta_2(l,w)\rangle \Leftarrow \mathsf{fn}^{\mathsf{i}}(xs) \twoheadrightarrow \langle ys, w\rangle \\
\mathsf{fn}^{\mathsf{i}}(\mathsf{city}(c):xs) \to \langle ys, \beta_3(c,w)\rangle \Leftarrow \mathsf{fn}^{\mathsf{i}}(xs) \twoheadrightarrow \langle ys, w\rangle
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$$

$$
\begin{array}{l}
\mathsf{fn}^{-1}([\,], \beta_1) \to [\,] \\
\mathsf{fn}^{-1}(n:ys, \beta_2(l,w)) \to \mathsf{person}(n,l):xs \Leftarrow \mathsf{fn}^{-1}(ys,w) \twoheadrightarrow xs \\
\mathsf{fn}^{-1}(ys, \beta_3(c,w)) \to \mathsf{city}(c):xs \Leftarrow \mathsf{fn}^{-1}(ys,w) \twoheadrightarrow xs
\end{array}
$$

Generation of a “put” function (given a “get” function f):

$$
\mathsf{put}_{\mathsf{f}}(v,s) \to s' \Leftarrow \mathsf{f}^{\mathsf{i}}(s) \twoheadrightarrow \langle \_, \pi\rangle, \mathsf{f}^{-1}(v,\pi) \twoheadrightarrow s'
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Stepwise approach to bidirectionalization

$$\mathsf{fn}^{\mathsf{i}}([\,]) \to \langle [\,], \beta_1 \rangle$$
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$$\mathsf{fn}^{\mathsf{i}}(\mathsf{city}(c) : xs) \to \langle ys, \beta_3(c, w) \rangle \Leftarrow \mathsf{fn}^{\mathsf{i}}(xs) \twoheadrightarrow \langle ys, w \rangle$$

$$\mathsf{fn}^{-1}([\,], \beta_1) \to [\,]$$
$$\mathsf{fn}^{-1}(n : ys, \beta_2(l, w)) \to \mathsf{person}(n, l) : xs \Leftarrow \mathsf{fn}^{-1}(ys, w) \twoheadrightarrow xs$$
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1. compute the complement of the original source

Stepwise approach to bidirectionalization

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\begin{array}{l}
\mathsf{fn}^{\mathsf{i}}([\,]) \to \langle [\,], \beta_1 \rangle \\
\mathsf{fn}^{\mathsf{i}}(\mathsf{person}(n, l) : xs) \to \langle n : ys, \beta_2(l, w) \rangle \Leftarrow \mathsf{fn}^{\mathsf{i}}(xs) \twoheadrightarrow \langle ys, w \rangle \\
\mathsf{fn}^{\mathsf{i}}(\mathsf{city}(c) : xs) \to \langle ys, \beta_3(c, w) \rangle \Leftarrow \mathsf{fn}^{\mathsf{i}}(xs) \twoheadrightarrow \langle ys, w \rangle
\end{array}
$$

$$
\begin{array}{l}
\mathsf{fn}^{-1}([\,], \beta_1) \to [\,] \\
\mathsf{fn}^{-1}(n : ys, \beta_2(l, w)) \to \mathsf{person}(n, l) : xs \Leftarrow \mathsf{fn}^{-1}(ys, w) \twoheadrightarrow xs \\
\mathsf{fn}^{-1}(ys, \beta_3(c, w)) \to \mathsf{city}(c) : xs \Leftarrow \mathsf{fn}^{-1}(ys, w) \twoheadrightarrow xs
\end{array}
$$

Generation of a “put” function (given a “get” function f):

$$
\mathsf{put}_{\mathsf{f}}(v, s) \to \boxed{s'} \Leftarrow \mathsf{f}^{\mathsf{i}}(s) \twoheadrightarrow \langle \_, \pi \rangle, \boxed{\mathsf{f}^{-1}(v, \pi) \twoheadrightarrow s'}
$$

1. compute the complement of the original source
2. compute the updated source

Limitations

Given, $s =$
[person(john, smith), city(london), person(ada, lovelace)]
and
$\mathrm{fn}^{\mathrm{i}}(s) = \langle [\mathrm{john}, \mathrm{ada}], \underbrace{\beta_2(\mathrm{smith}, \beta_3(\mathit{london}, \beta_2(\mathit{lovelace}, \beta_1)))}_{\pi}\rangle$

Update 1 (compatible)

$[\mathrm{john}, \mathrm{ada}] \Rightarrow [\mathrm{peter}, \mathrm{ada}]\ (v_1)$
$\mathrm{fn}^{-1}(v_1, \pi) =$
[person(peter, smith), city(london), person(ada, lovelace)]

Update 2 (non-compatible)

$[\mathrm{john}, \mathrm{ada}] \Rightarrow [\mathrm{john}]\ (v_2)$
$\mathrm{fn}^{-1}(v_2, \pi)$ undefined

Limitations

Given, $s =$
$[\mathsf{person}(\mathsf{john},\mathsf{smith}),\mathsf{city}(\mathsf{london}),\mathsf{person}(\mathsf{ada},\mathsf{lovelace})]$
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Characterizing compatible updates [RC 2019]

Definition (view skeleton)

Consider a source s with $\mathsf{f}^{\mathsf{i}}(s) = \langle v, \pi \rangle$

We compute the narrowing derivation:
$\mathsf{f}^{-1}(x, \pi) \leadsto^{*}_{\sigma} s'$ (deterministic!)

Then, $x\sigma = v'$ is the view skeleton

E.g.,
$\mathsf{fn}^{-1}(x, \beta_2(\mathsf{smith}, \beta_3(\mathit{london}, \beta_2(\mathit{lovelace}, \beta_1)))) \leadsto^{*}_{\{x \mapsto [x_1, x_2]\}} s'$
Therefore, the view skeleton is $[x_1, x_2]$

Consider a source s with $\mathsf{f}^{\mathsf{i}}(s) = \langle v, \pi \rangle$

An update v' is **compatible** if it is an instance of the view skeleton

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Consider a source s with $\mathsf{f}^{\mathsf{i}}(s) = \langle v, \pi \rangle$

An update v' is **compatible** if it is an instance of the view skeleton

Ongoing work: non-compatible updates...

Concurrent Programming

A first-order, eager functional and concurrent language based on message-passing

The language

We consider a simple functional and concurrent programming language similar to **Erlang**

- No shared memory, only message passing (asynchronous communication)
- Each process has a pid and a local queue (mailbox)
- A system is a collection of processes

Sequential Erlang in 5 examples

append/2

```
append([H|T], L) -> [H|append(T,L)];
append([],    L) -> L.
```

Variables start with an uppercase letter

Function names and atoms (i.e., constants) start with a lowercase letter

Alternative definition:

append/2

```
append(A,B) -> case A of
                 [H|T]   -> [H|append(T,L)];
                 []      -> L
               end.
```

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```

Sequential Erlang in 5 examples

toint/1

```
toint({s,N})  ->  int(N)+1;
toint(zero)   ->  0.
```

E.g., `toint({s,{s,{s,zero}}})` evaluates to `3`

No user-defined algebraic data types (so we cannot write `s(s(s(zero)))`)

Main data types: numbers, atoms, lists, and tuples

Sequential Erlang in 5 examples

factorial/1

```
factorial(N) when N > 0  ->  N * factorial(N - 1);
factorial(1)             ->  0.
```

Besides pattern matching, we can have guards

Only built-in functions are allowed in guards

Sequential Erlang in 5 examples

minmax/1

$$\begin{array}{lll} \texttt{minmax(L)} & \texttt{->} & \texttt{Min} = \texttt{lists:min(L)}, \\ & & \texttt{Max} = \texttt{lists:max(L)}, \\ & & \{\texttt{Min},\texttt{Max}\}. \end{array}$$

Sequence $e_1, \ldots, e_n$ evaluates all expressions, returns the evaluation of e_n

Equation $\texttt{pat} = \texttt{exp}$ evaluates $\texttt{exp}$ and perform pattern matching with $\texttt{pat}$

Equivalent to

$$\texttt{minmax(L)} \texttt{->} \{\texttt{Min},\texttt{Max}\} \Leftarrow \texttt{lists:min(L)} \twoheadrightarrow \texttt{Min}, \; \texttt{lists:max(L)} \twoheadrightarrow \texttt{Max}$$

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Sequential Erlang in 5 examples

inclist/1

```
inclist(L) -> lists:map(fun(X) -> X + 1 end, L).
```

Higher-order functions

Anonymous functions

No partial applications

Concurrency features

- spawn: creates a new process as a side-effect and returns the pid of the new process
- self: returns the pid of the current process
- pid ! val: sends val to process pid as a side-effect and returns val
- receive ... end: waits for a message that matches some pattern (otherwise, blocks execution) and returns the expression in the selected branch

Concurrent Erlang in 1 example


```
main()      ->  S = spawn(server([])),
                client(S).

client(S)   ->  S ! {self(), {add, paper}},
                S ! {self(), {add, pencil}},
                S ! {self(), take},
                receive
                  X -> X
                end.

server(L)   ->  receive
                  {_, {add, Item}} -> server([Item|L]);
                  {C, take} -> C ! hd(L), server(tl(L))
                end.
```

From Erlang to Core Erlang

Core Erlang is an intermediate representation used during the compilation of Erlang programs

It is a convenient representation for defining analyses and other tools

Not as readable as Erlang. . .

From Erlang to Core Erlang

erlang

```
a(42)  ->  ok;
a(N)   ->  M = N + 1, a(M).
```

core erlang

```
'a'/1 = fun(_@c0) ->
    case _@c0 of
      <42> when 'true' -> 'ok'
      <_@c2> when 'true' -> let <_@c3> = call 'erlang':'+'(N, 1)
                              in apply 'a'/1 (_@c3)
    end
```

Essentially: one clause per function, case for pattern matching, let for sequences, apply for function applications, ...

From Erlang to Core Erlang

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Essentially: one clause per function, case for pattern matching, let for sequences, apply for function applications, ...

Core Erlang syntax

We consider a subset of Core Erlang with this syntax:

$$
\begin{array}{rcl}
\mathit{Module} & ::= & \mathrm{module}\ \mathit{Atom} = \mathit{fun}_1, \ldots, \mathit{fun}_n \\
\mathit{fun} & ::= & \mathit{fname} = \mathrm{fun}\ (X_1, \ldots, X_n) \to \mathit{expr} \\
\mathit{fname} & ::= & \mathit{Atom}/\mathit{Integer} \\
\mathit{lit} & ::= & \mathit{Atom} \mid \mathit{Integer} \mid \mathit{Float} \mid [\,] \\
\mathit{expr} & ::= & \mathit{Var} \mid \mathit{lit} \mid \mathit{fname} \mid [\mathit{expr}_1|\mathit{expr}_2] \mid \{\mathit{expr}_1, \ldots, \mathit{expr}_n\} \\
& \mid & \mathrm{call}\ \mathit{expr}\ (\mathit{expr}_1, \ldots, \mathit{expr}_n) \mid \mathrm{apply}\ \mathit{expr}\ (\mathit{expr}_1, \ldots, \mathit{expr}_n) \\
& \mid & \mathrm{case}\ \mathit{expr}\ \mathrm{of}\ \mathit{clause}_1; \ldots; \mathit{clause}_m\ \mathrm{end} \\
& \mid & \mathrm{let}\ \mathit{Var} = \mathit{expr}_1\ \mathrm{in}\ \mathit{expr}_2 \mid \mathrm{receive}\ \mathit{clause}_1; \ldots; \mathit{clause}_n\ \mathrm{end} \\
& \mid & \mathrm{spawn}(\mathit{expr}, [\mathit{expr}_1, \ldots, \mathit{expr}_n]) \mid \mathit{expr}_1\ !\ \mathit{expr}_2 \mid \mathrm{self}() \\
\mathit{clause} & ::= & \mathit{pat}\ \mathrm{when}\ \mathit{expr}_1 \to \mathit{expr}_2 \\
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\end{array}
$$

Some preliminary definitions

Definition (process) //no local queue!

A process is a triple $\langle p, \theta, e \rangle$ where

- p is the pid of the process
- θ is an environment
- e is the expression to be reduced

Definition (system)

A system is denoted by $\Gamma; \Pi$, where

- Γ models the network & local queues (global mailbox); a multiset of triples $(sender\_pid, target\_pid, message)$
- Π is a pool of processes

We use $\boxed{\Gamma; \langle p, \theta, e \rangle \ \& \ \Pi}$ *to denote an arbitrary system*

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Semantics

Erlang guarantees that, if two messages are sent from process p to process p', and both are delivered, then the order of these messages is kept

1. [LOPSTR16] ensures this restriction
2. [JLAMP18,FLOPS18,FORTE19] ignore this restriction

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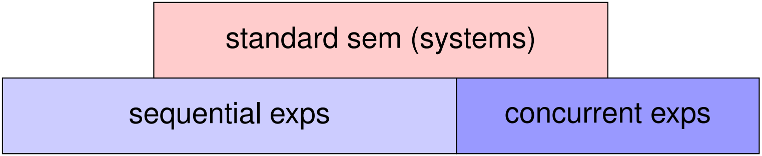
Reduction semantics (layers)



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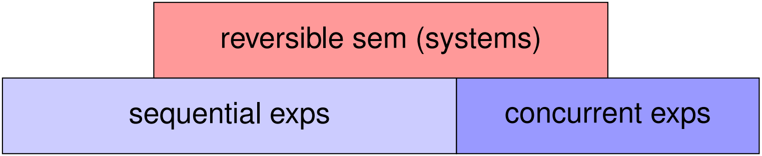

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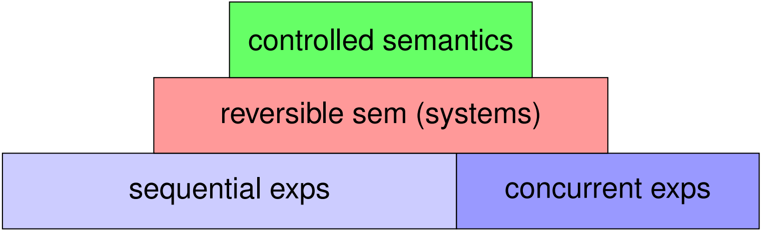

Semantics of concurrent expressions

For concurrent actions, we face the following problems:

1. we don't know the result of the actions (fresh variables)
2. we must perform side effects (labels)

Labels

- At expression level, transitions for concurrent actions are labelled with enough information
- At system level, labels are used to perform the associated actions

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Expression semantics: sequential expressions

$$(\mathit{Var})\ \frac{}{\theta, X \xrightarrow{\tau} \theta, \theta(X)} \qquad (\mathit{Tuple})\ \frac{\theta, e_i \xrightarrow{\ell} \theta', e_i'}{\theta, \{\overline{v_{1,i-1}}, e_i, \overline{e_{i+1,n}}\} \xrightarrow{\ell} \theta', \{\overline{v_{1,i-1}}, e_i', \overline{e_{i+1,n}}\}}$$

$$(\mathit{List1})\ \frac{\theta, e_1 \xrightarrow{\ell} \theta', e_1'}{\theta, [e_1 | e_2] \xrightarrow{\ell} \theta', [e_1' | e_2]} \qquad (\mathit{List2})\ \frac{\theta, e_2 \xrightarrow{\ell} \theta', e_2'}{\theta, [v_1 | e_2] \xrightarrow{\ell} \theta', [v_1 | e_2']}$$

$$(\mathit{Let1})\ \frac{\theta, e_1 \xrightarrow{\ell} \theta', e_1'}{\theta, \mathsf{let}\ X = e_1\ \mathsf{in}\ e_2 \xrightarrow{\ell} \theta', \mathsf{let}\ X = e_1'\ \mathsf{in}\ e_2} \qquad (\mathit{Let2})\ \frac{}{\theta, \mathsf{let}\ X = v\ \mathsf{in}\ e \xrightarrow{\tau} \theta[X \mapsto v], e}$$

$$(\mathit{Case1})\ \frac{\theta, e \xrightarrow{\ell} \theta', e'}{\begin{array}{l}\theta, \mathsf{case}\ e\ \mathsf{of}\ cl_1; \ldots; cl_n\ \mathsf{end} \\ \xrightarrow{\ell} \theta', \mathsf{case}\ e'\ \mathsf{of}\ cl_1; \ldots; cl_n\ \mathsf{end}\end{array}} \qquad (\mathit{Case2})\ \frac{\mathsf{match}(v, cl_1, \ldots, cl_n) = \langle \theta_i, e_i \rangle}{\theta, \mathsf{case}\ v\ \mathsf{of}\ cl_1; \ldots; cl_n\ \mathsf{end} \xrightarrow{\tau} \theta\theta_i, e_i}$$

$$(\mathit{Apply1})\ \frac{\theta, e_i \xrightarrow{\ell} \theta', e_i' \quad i \in \{1, \ldots, n\}}{\theta, \mathsf{apply}\ a/n\ (\overline{v_{1,i-1}}, e_i, \overline{e_{i+1,n}}) \xrightarrow{\ell} \theta', \mathsf{apply}\ a/n\ (\overline{v_{1,i-1}}, e_i', \overline{e_{i+1,n}})}$$

$$(\mathit{Apply2})\ \frac{\mu(a/n) = \mathsf{fun}\ (X_1, \ldots, X_n) \to e}{\theta, \mathsf{apply}\ a/n\ (v_1, \ldots, v_n) \xrightarrow{\tau} \{X_1 \mapsto v_1, \ldots, X_n \mapsto v_n\}, e}$$

Sending a message

(expression semantics)

$$(\mathit{Send1})\ \frac{\theta, e_1 \xrightarrow{\ell} \theta', e_1'}{\theta, e_1 \,!\, e_2 \xrightarrow{\ell} \theta', e_1' \,!\, e_2} \qquad \frac{\theta, e_2 \xrightarrow{\ell} \theta', e_2'}{\theta, v_1 \,!\, e_2 \xrightarrow{\ell} \theta', v_1 \,!\, e_2'}$$

$$(\mathit{Send2})\ \frac{}{\theta, v_1 \,!\, v_2 \xrightarrow{\mathsf{send}(v_1, v_2)} \theta, v_2}$$

(system semantics)

$$(\mathit{Send})\ \frac{\theta, e \xrightarrow{\mathsf{send}(p', v)} \theta', e'}{\Gamma; \langle p, \theta, e\rangle \& \Pi \hookrightarrow \Gamma \cup \{(p, p', v)\}; \langle p, \theta', e'\rangle \& \Pi}$$

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Self

(expression semantics)

$$(\mathit{Self})\ \frac{}{\theta, \mathsf{self}() \xrightarrow{\mathit{self}(\kappa)} \theta, \kappa}$$

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Spawning a process

(expression semantics)

$$(\mathit{Spawn})\ \frac{}{\theta, \mathsf{spawn}(a/n, [v_1, \ldots, v_n]) \xrightarrow{\mathsf{spawn}(\kappa, a/n, [\overline{v_n}])} \theta, \kappa}$$

(system semantics)

$$(\mathit{Spawn})\ \frac{\theta, e \xrightarrow{\mathsf{spawn}(\kappa, a/n, [\overline{v_n}])} \theta', e' \quad p' \text{ is a fresh pid}}{\Gamma; \langle p, \theta, e\rangle \& \Pi \hookrightarrow \Gamma; \langle p, \theta', e'\{\kappa \mapsto p'\}\rangle \& \langle p', \theta', \mathsf{apply}\ a/n\ (\overline{v_n})\rangle \& \Pi}$$

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Receiving a message

(expression semantics)

$$(\textit{Receive})\ \frac{}{\theta, \mathsf{receive}\ cl_1; \ldots; cl_n\ \mathsf{end} \xrightarrow{\mathsf{rec}(\kappa, \overline{cl_n})} \theta, \kappa}$$

(system semantics)

$$(\textit{Receive})\ \frac{\theta, e \xrightarrow{\mathsf{rec}(\kappa, \overline{cl_n})} \theta', e' \quad \mathsf{matchrec}(\theta, \overline{cl_n}, v) = (\theta_i, e_i)}{\Gamma \cup \{(p', p, v)\}; \langle p, \theta, e\rangle \& \Pi \hookrightarrow \Gamma; \langle p, \theta'\theta_i, e'\{\kappa \mapsto e_i\}\rangle \& \Pi}$$

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Reversible semantics (uncontrolled)

1. Forward reversible semantics: we instrument the system rules using a Landauer embedding
2. Backward reversible semantics: straightforward inversion of the previous rules

Processes have now the form $\langle p, h, \theta, e \rangle$

history h

is a sequence of terms headed by constructors seq, send, rec, spawn, and self, and whose arguments are the information required to (deterministically) undo the step

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Uncontrolled forward semantics

(Send)

$$\frac{\theta, e \xrightarrow{\mathsf{send}(p',v)} \theta', e'}{\begin{array}{l}\Gamma; \langle p, h, \theta, e\rangle \mid \Pi \\ \rightharpoonup_{p,\mathsf{send}(\ell)} \Gamma \cup \{(p, p', v)\}; \\ \qquad \langle p, \mathsf{send}(\theta, e, p', v) : h, \theta', e'\rangle \mid \Pi\end{array}}$$

$(\mathit{Receive})$

$$\frac{\theta, e \xrightarrow{\mathsf{rec}(\kappa,\overline{cl_n})} \theta', e' \ \text{ and } \ \mathsf{matchrec}(\theta, \overline{cl_n}, v) = (\theta_i, e_i)}{\begin{array}{l}\Gamma \cup \{(p', p, v)\}\langle p, h, \theta, e\rangle \mid \Pi \\ \rightharpoonup_{p,\mathsf{rec}(\ell)} \Gamma; \langle p, \mathsf{rec}(\theta, e, p', v) : h, \theta'\theta_i, e'\{\kappa \mapsto e_i\}\rangle \mid \Pi\end{array}}$$

(Spawn)

$$\frac{\theta, e \xrightarrow{\mathsf{spawn}(\kappa,a/n,[\overline{v_n}])} \theta', e' \quad \text{and } p' \text{ is a fresh pid}}{\begin{array}{l}\Gamma; \langle p, h, \theta, e\rangle \mid \Pi \\ \rightharpoonup_{p,\mathsf{spawn}(p')} \Gamma; \langle p, \mathsf{spawn}(\theta, e, p') : h, \theta', e'\{\kappa \mapsto p'\}\rangle \\ \qquad \mid \langle p', [\,], id, \mathsf{apply}\ a/n\ (\overline{v_n})\rangle \mid \Pi\end{array}}$$

Uncontrolled backward semantics

$$(\overline{Send})\quad \begin{array}{l} \Gamma \cup \{(p, p', v)\}; \langle p, \mathsf{send}(\theta, e, p', v) : h, \theta', e'\rangle \mid \Pi \\ \quad \leftharpoondown_{p,\mathsf{send}(\ell)} \Gamma; \langle p, h, \theta, e\rangle \mid \Pi \end{array}$$

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$\Rightarrow$ reversible computations must be **causal consistent**

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Designing a reversible debugger

A common problem: in concurrent languages, replaying a particular computation might be difficult (even impossible) given the nondeterminism of the language

Solution

1. instrument the code so that it generates a **log** (a sequence of messages received by each process)
2. forward reversible semantics is **driven** by the log (causal-consistent replay semantics)
3. **controlled** reversible semantics driven by user requests (both replay requests and rollbacks)

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Logging semantics

We tag messages with unique identifiers

$$v \quad \mapsto \quad \{v, \ell\}, \qquad \text{where } \ell \text{ is fresh}$$

A log $\mathcal{L}(d)$ of a derivation d is a sequence of items spawn(p), send(ℓ) or rec(ℓ) for each process in d

(logs are local to each process)

(*Seq*)

$$\frac{\theta, e \xrightarrow{\tau} \theta', e'}{\Gamma; \langle p, \theta, e\rangle \mid \Pi \hookrightarrow_{p,\mathsf{seq}} \Gamma; \langle p, \theta', e'\rangle \mid \Pi}$$

(*Send*)

$$\frac{\theta, e \xrightarrow{\mathsf{send}(p',v)} \theta', e' \text{ and } \ell \text{ is a fresh symbol}}{\Gamma; \langle p, \theta, e\rangle \mid \Pi \hookrightarrow_{p,\mathsf{send}(\ell)} \Gamma \cup \{(p, p', \{v, \ell\})\}; \langle p, \theta', e'\rangle \mid \Pi}$$

(*Receive*)

$$\frac{\theta, e \xrightarrow{\mathsf{rec}(\kappa,\overline{cl_n})} \theta', e' \quad \text{and} \quad \mathsf{matchrec}(\theta, \overline{cl_n}, v) = (\theta_i, e_i)}{\Gamma \cup \{(p', p, \{v, \ell\})\}; \langle p, \theta, e\rangle \mid \Pi \hookrightarrow_{p,\mathsf{rec}(\ell)} \Gamma; \langle p, \theta'\theta_i, e'\{\kappa \mapsto e_i\}\rangle \mid \Pi}$$

(*Spawn*)

$$\frac{\theta, e \xrightarrow{\mathsf{spawn}(\kappa,a/n,[\overline{v_n}])} \theta', e' \quad \text{and} \quad p' \text{ is a fresh pid}}{\Gamma; \langle p, \theta, e\rangle \mid \Pi \hookrightarrow_{p,\mathsf{spawn}(p')} \Gamma; \langle p, \theta', e'\{\kappa \mapsto p'\}\rangle \mid \langle p', id, \mathsf{apply}\ a/n\ (\overline{v_n})\rangle \mid \Pi}$$

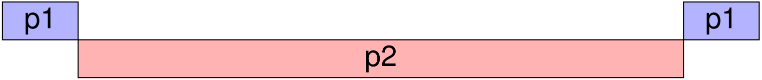
(*Self*)

$$\frac{\theta, e \xrightarrow{\mathsf{self}(\kappa)} \theta', e'}{\Gamma; \langle p, \theta, e\rangle \mid \Pi \hookrightarrow_{p,\mathsf{self}} \Gamma; \langle p, \theta', e'\{\kappa \mapsto p\}\rangle \mid \Pi}$$

(implemented by a program instrumentation)

Causally equivalent derivations

Traditional reversible debuggers allow us to go backwards in exactly the inverse order of the forward computation



If p1 and p2 are independent then

are **causally equivalent**

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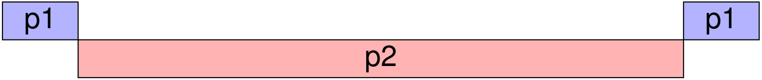

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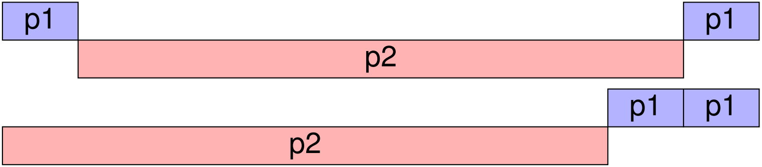

are **causally equivalent**

Causally equivalent derivations

d_1 and d_2 are causally equivalent ($d_1 \approx d_2$) if d_1 can be obtained from d_2 by switching consecutive transitions **as long as**

1. the actions of a given process cannot be switched
2. no message can be received before it is sent
3. a process cannot perform any action before it is spawned

Given (coinitial) derivations d_1 and d_2, $\boxed{\mathcal{L}(d_1) = \mathcal{L}(d_2) \text{ iff } d_1 \approx d_2}$

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Reversible Semantics

Processes have the form $\langle p, \omega, h, \theta, e \rangle$
with ω a *log* and h a *history*

A history h is a sequence of terms headed by constructors seq, send, rec, spawn, and self, and whose arguments are the information required to (deterministically) undo the step

Uncontrolled forward semantics

(Send)

$$\frac{\theta, e \xrightarrow{\mathsf{send}(p',v)} \theta', e'}{\begin{array}{l}\Gamma; \langle p, \mathsf{send}(\ell)\!:\!\omega, h, \theta, e\rangle \mid \Pi \\ \rightharpoonup_{p,\mathsf{send}(\ell),\{\mathsf{s},\ell\Uparrow\}} \Gamma \cup \{(p, p', \{v,\ell\})\}; \\ \qquad \langle p, \omega, \mathsf{send}(\theta, e, p', \{v,\ell\})\!:\!h, \theta', e'\rangle \mid \Pi\end{array}}$$

$(\mathit{Receive})$

$$\frac{\theta, e \xrightarrow{\mathsf{rec}(\kappa,\overline{cl_n})} \theta', e' \;\text{ and }\; \mathsf{matchrec}(\theta, \overline{cl_n}, v) = (\theta_i, e_i)}{\begin{array}{l}\Gamma \cup \{(p', p, \{v,\ell\})\}\langle p, \mathsf{rec}(\ell)\!:\!\omega, h, \theta, e\rangle \mid \Pi \\ \rightharpoonup_{p,\mathsf{rec}(\ell),\{\mathsf{s},\ell\Downarrow\}} \Gamma; \langle p, \omega, \mathsf{rec}(\theta, e, p', \{v,\ell\})\!:\!h, \theta'\theta_i, e'\{\kappa \mapsto e_i\}\rangle \mid \Pi\end{array}}$$

(Spawn)

$$\frac{\theta, e \xrightarrow{\mathsf{spawn}(\kappa,a/n,[\overline{v_n}])} \theta', e' \;\text{ and }\; \omega' = \mathsf{trace}(d, p')}{\begin{array}{l}\Gamma; \langle p, \mathsf{spawn}(p')\!:\!\omega, h, \theta, e\rangle \mid \Pi \\ \rightharpoonup_{p,\mathsf{spawn}(p'),\{\mathsf{s},\mathsf{sp}_{p'}\}} \Gamma; \langle p, \omega, \mathsf{spawn}(\theta, e, p')\!:\!h, \theta', e'\{\kappa \mapsto p'\}\rangle \\ \qquad \mid \langle p', \omega', [\,], id, \mathsf{apply}\ a/n\ (\overline{v_n})\rangle \mid \Pi\end{array}}$$

Uncontrolled backward semantics

$$(\overline{Send}) \quad \begin{array}{l} \Gamma \cup \{(p, p', \{v, \ell\})\}; \langle p, \omega, \mathsf{send}(\theta, e, p', \{v, \ell\}) : h, \theta', e' \rangle \mid \Pi \\ \leftharpoondown_{p,\mathsf{send}(\ell),\{\mathsf{s},\ell\Uparrow\}} \; \Gamma; \langle p, \mathsf{send}(\ell) : \omega, h, \theta, e \rangle \mid \Pi \end{array}$$

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Some results...

Coinitial derivations are cofinal
iff they are causally equivalent

Misbehaviors are preserved
by all causally equivalent derivations

Controlled replay/rollback semantics

We allow the user to start a replay/rollback until a particular action is performed, e.g.,

- $\{p, \mathsf{s}\}$: one step backward/forward of process p
- $\{p, \ell^{\Uparrow}\}$: a backward/forward derivation of process p up to the sending of the message tagged with ℓ
- $\{p, \ell^{\Downarrow}\}$: a backward/forward derivation of process p up to the reception of the message tagged with ℓ
- $\{p, \mathsf{sp}_{p'}\}$: a backward/forward derivation of process p up to the spawning of the process with pid p'
- $\{p, X\}$: a backward derivation of process p up to the introduction of variable X
- …

Controlled semantics takes a stack of requests (initially one)

It is defined as a layer on top of the uncontrolled semantics:

- If a process can perform a step satisfying the request on top of the stack → do it and remove the request
- If a process can perform a step but it doesn't satisfy the request → update the system but keep the request
- If a step on the process is not possible → track dependencies and add new requests on top of the stack

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Reversible debugging

Two components: code instrumentation (logging)
$+$ causal-consistent reversible debugger (CauDEr)

`https://github.com/mistupv/tracer`
`https://github.com/mistupv/cauder/tree/replay`

A Note on the implementation

Current prototypes show good potential, but more implementation effort is still required:

- move from Core Erlang to Erlang
- graphical representation of logs
- consider more Erlang features: links, monitors, built-in's, input/output, behaviours, etc
- combine it with program slicing / automatic bug location

Recap

Every (irreversible) language can be made reversible by defining a Landauer embedding

Recap

Reversibilization can be achieved by instrumenting the semantics or the program

Recap

Reversibilization $\neq$ program inversion

Recap

For concurrent languages, causal consistency is essential

Recap

There are many other applications of reversible computation: quantum computing, discrete simulation, hardware design, computational biology, robotics, etc

Thanks for your attention!

Questions?