Fast Offline Partial Evaluation of Large Logic Programs

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Introduction

context: offline partial evaluation of logic programs

- PE = BTA + specialization
- BTA = termination analysis + propagation of BTs + annotation

Annotations

- calls are annotated as unfold/memo
- arguments are annotated as static/dynamic

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specialization

1. take an atom and unfold it as much as possible (following the unfold/memo annotations) — local control
2. for every atom in the leaves — global control
   - generalize arguments marked as dynamic
   - add it to the set of atoms to be partially evaluated
3. go to step (1)

Termination issues in PE are classified into

- **local termination:** no atom is infinitely unfolded
- **global termination:** no infinitely many atoms are unfolded

Annotations are **safe** when:

- unfold/memo annotations guarantee local termination
- static/dynamic annotations guarantee global termination
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Fully automatic BTA for logic programs
[Craig, Gallagher, Leuschel, Henriksen, LOPSTR 2004]

- current implementation does not guarantee global termination
- complex design (different analyses running on different Prolog systems) $\rightarrow$ difficult to maintain
- slow, does not scale to medium-sized examples

In this paper, we propose a new BTA:

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Termination analysis

Essential component of BTA

Choice:

dependent of a computation rule

$$H \leftarrow B_1, B_2, \ldots, B_n \qquad H \leftarrow B_1, B_2, \ldots, B_n$$

- more accurate
- slower (requires reexecution every time an annotation changes)

independent of a computation rule

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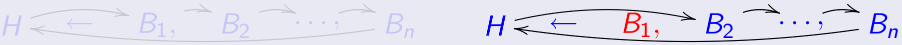

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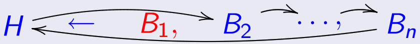

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Quasi-termination analysis for logic programs [Vidal, PEPM 2007]

- considers **strong** termination (independent of the computation rule)
- based on size-change analysis [Lee, Jones, Ben-Amram, POPL 2001]
- covers termination (unfold/memo) and quasi-termination (static/dynamic)

size-change analysis

1. construction of size-change graphs
2. computation of transitive closure (by concatenation)
3. identification of program loops (maximal graphs)

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Construction of size-change graphs

Size-change graphs are used to trace size changes of predicate arguments from one call to another

Parameterized by a **reduction pair** $(\succsim, \succ)$

1. $\succsim$ is a quasi-order [reflexive & transitive]
2. $\succ$ is a well-founded order [irreflexive & transitive]
3. $\succsim$ and $\succ$ are closed under substitutions $[s \succsim t \;\Rightarrow\; \sigma(s) \succsim \sigma(t)]$
4. they are compatible

   (i.e., $\succsim \circ \succ \subseteq \succ$ and $\succ \circ \succsim \subseteq \succ$ but $\succsim \subseteq \succ$ is not necessary)

which can be induced from a **symbolic norm**

$$s \succ t \;\Leftrightarrow\; ||s|| > ||t|| \quad \text{and} \quad s \succsim t \;\Leftrightarrow\; ||s|| \geqslant ||t||$$

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Symbolic norms

symbolic term-size norm

$$||t||_{ts} = \begin{cases} n + \sum_{i=0}^{n} ||t_i||_{ts} & \text{if } t = f(t_1, \ldots, t_n),\ n \geqslant 0 \\ t & \text{if } t \text{ is a variable} \end{cases}$$

E.g., $||\mathbf{f}(\mathbf{a}, \mathbf{b})||_{ts} = \mathbf{2}$, but $||\mathbf{f}(\mathbf{X}, \mathbf{Y})||_{ts} = \mathbf{2} + \mathbf{X} + \mathbf{Y}$

symbolic list-length norm

$$||t||_{ll} = \begin{cases} 1 + ||Xs||_{ll} & \text{if } t = [X|Xs] \\ t & \text{if } t \text{ is a variable} \\ 0 & \text{otherwise} \end{cases}$$

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Example: construction of size-change graphs

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\begin{array}{lll}
\mathbf{incList}([\,], \_, [\,]). & & \\
\mathbf{incList}([\mathbf{X}|\mathbf{R}], \mathbf{I}, \mathbf{L}) & \leftarrow & \mathbf{iList}(\mathbf{X}, \mathbf{R}, \mathbf{I}, \mathbf{L}). \\
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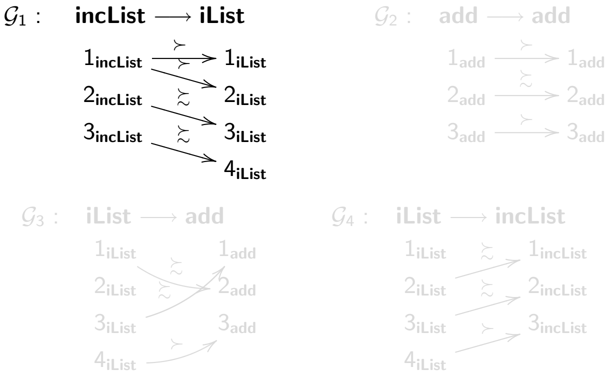

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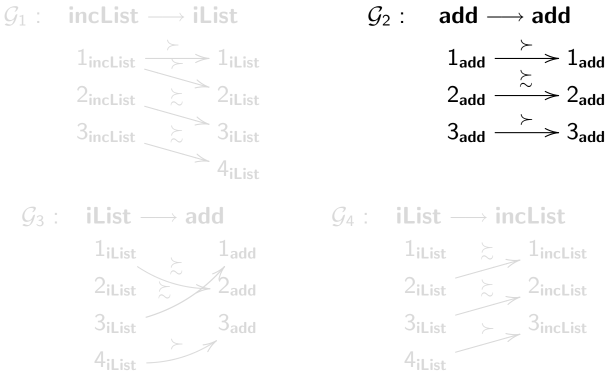

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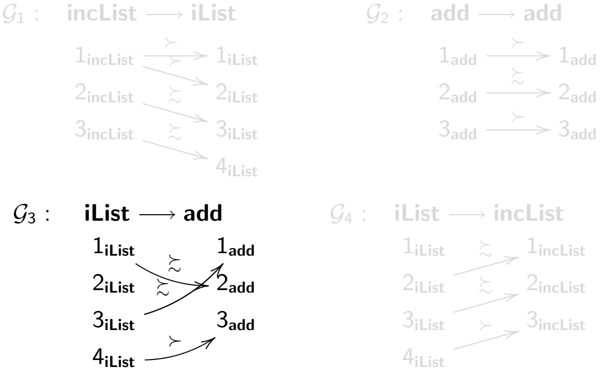

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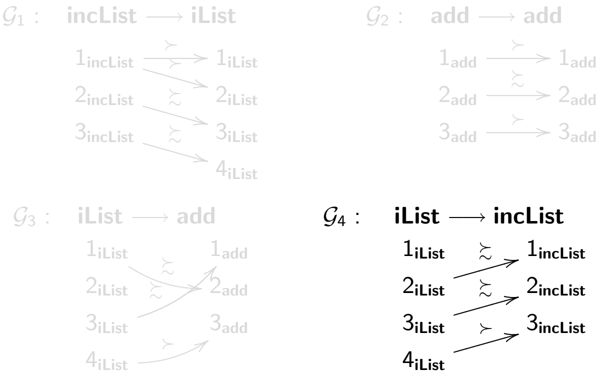

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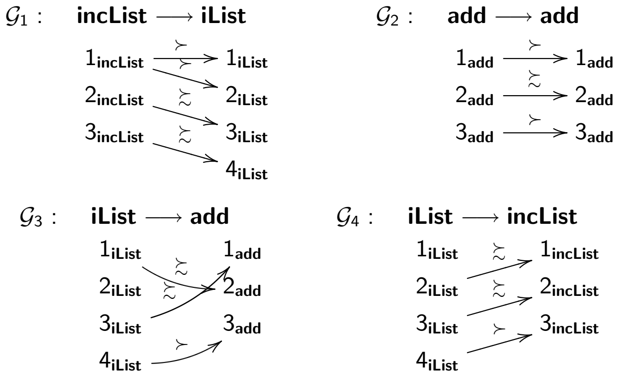
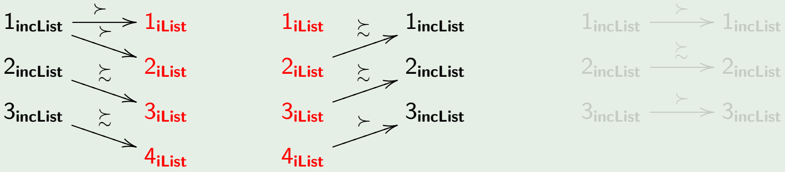


Transitive closure:

- compute all possible concatenations of graphs

Example

$\mathcal{G}_1$:**incList** $\longrightarrow$ **iList** $\bullet$ $\mathcal{G}_4$:**iList** $\longrightarrow$ **incList** $=$ $\mathcal{G}_{14}$:**incList** $\longrightarrow$ **incList**



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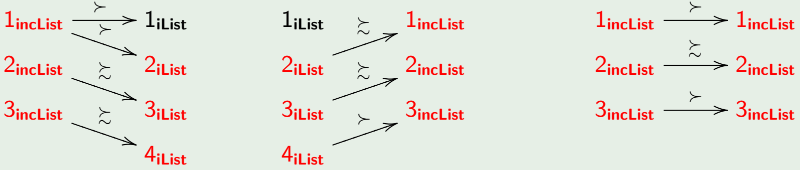

Identification of program loops

maximal graph

A size-change graph G is maximal if

1. its input and output nodes are the same
2. it is idempotent, i.e., $G = G \cdot G$

maximal graph $\approx$ program loop

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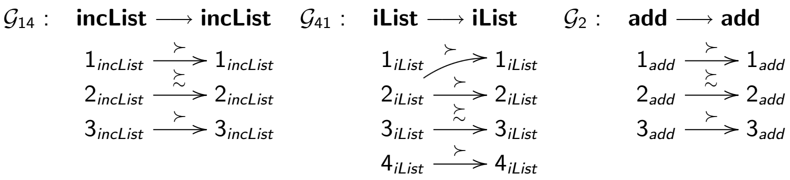

Size-change terminating! (acc. to [Lee, Jones, Ben-Amram, POPL 2001])

Too weak in logic programming...

$$\mathbf{add}(\mathbf{X},\mathbf{Y},\mathbf{Z}) \Rightarrow_{\{X \mapsto s(X')],\ Z \mapsto s(Z')\}} \mathbf{add}(\mathbf{X}',\mathbf{Y},\mathbf{Z}') \Rightarrow_{\{X' \mapsto s(X''),\ Z' \mapsto s(Z'')\}} \mathbf{add}(\mathbf{X}'',\mathbf{Y},\mathbf{Z}'') \Rightarrow \infty$$

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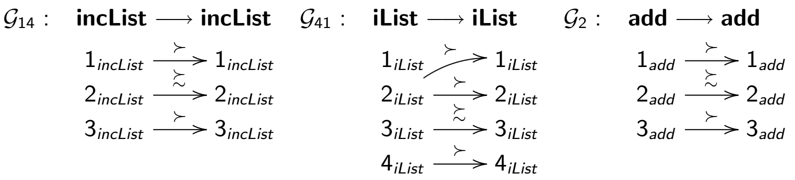

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Local termination

instantiated enough [Lindenstrauss, Sagiv, ICLP 1997]

Term t is instantiated enough w.r.t. $||\cdot||$ if $||t||$ is an integer

sufficient condition for termination

If every maximal graph for P contains at least one edge

$$i_p \stackrel{\succ}{\longrightarrow} i_p$$

such that, in every possible call, $p(t_1, \ldots, t_n)$, the argument t_i is instantiated enough w.r.t. $||\cdot||$, then P is terminating

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Global termination

Must ensure quasi-termination (only finitely many different atoms)

In our previous approach, global termination is ensured when

- for every predicate *p* and for every maximal graph for *p*, either
  - the previous condition hold or
  - there is an edge to every argument
- and the considered symbolic norm is **bounded**

  i.e., the set $\{s \mid ||t|| \geqslant ||s||\}$ is finite for any term t

with 1_{add} or 2_{add} static

But the symbolic list-length norm is not bounded...

$||[a]|| = ||[s(a)]|| = ||[s(s(a))]|| = \ldots = 1$

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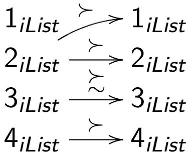

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  - the previous condition hold or
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Global termination

Must ensure quasi-termination (only finitely many different atoms)

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Is boundedness really needed?

Alternative approach

- use any symbolic norm (even if non-bounded)
- generalize problematic parts of atoms before adding them to the set of atoms to be partially evaluated (global level)

mgg: most general generalization of an atom w.r.t. a norm

$mgg^{||\cdot||}(t) = t'$

if t is the most general generalization of t such that $||t|| = ||mgg^{||\cdot||}(t)||$

$mgg^{||\cdot||_{ll}}([X]) = [X]$ $\quad$ $mgg^{||\cdot||_{ll}}([X,Y]) = [X,Y]$

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annotations for global termination

given a predicate *p* and an argument *i*:

1. if every maximal graph for *p* fulfills the local termination condition, i_p is marked as **static**
2. otherwise, if ∃ a maximal graph with no input edge to i_p, then it is marked as **dynamic**

Furthermore, arguments marked as **static** are changed to **list(dynamic)** if the list-length norm was used

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Example

$$\begin{array}{ll}
\mathbf{incList}([\,], _, [\,]). & \mathbf{add}(\mathbf{0}, \mathbf{Y}, \mathbf{Y}). \\
\mathbf{incList}([\mathbf{X}|\mathbf{R}], \mathbf{I}, \mathbf{L}) \leftarrow \mathbf{iList}(\mathbf{X}, \mathbf{R}, \mathbf{I}, \mathbf{L}). & \mathbf{add}(\mathbf{s}(\mathbf{X}), \mathbf{Y}, \mathbf{s}(\mathbf{Z})) \leftarrow \mathbf{add}(\mathbf{X}, \mathbf{Y}, \mathbf{Z}).
\end{array}$$

$$\begin{array}{l}
\mathbf{iList}(\mathbf{X}, \mathbf{R}, \mathbf{I}, [\mathbf{XI}|\mathbf{RI}]) \leftarrow \mathbf{add}(\mathbf{I}, \mathbf{X}, \mathbf{XI}), \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathbf{incList}(\mathbf{R}, \mathbf{I}, \mathbf{RI}).
\end{array}$$

1. User's input: *incList(dynamic, static, dynamic)*
2. Propagation of BTs: *incList(D, S, D)*, *iList(D, D, S, D)*, *add(S, D, D)*
3. size-change analysis:
   - local termination: *incList* ↦ *memo*, *iList* ↦ *memo*, *add* ↦ *unfold*
   - global termination: no **static**→**dynamic** change required

Example

$$\textbf{incList}([\,], \_, [\,]).$$
$$\textbf{incList}([X|R], I, L) \leftarrow \textbf{iList}(X, R, I, L).$$
$$\textbf{iList}(X, R, I, [XI|RI]) \leftarrow \textbf{add}(I, X, XI),\ \textbf{incList}(R, I, RI).$$
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incList([ ], _, [ ]).
incList([**X**|**R**], **I**, **L**) ←**iList**(**X**, **R**, **I**, **L**).

iList(**X**, **R**, **I**, [**XI**|**RI**])←**add**(**I**, **X**, **XI**),
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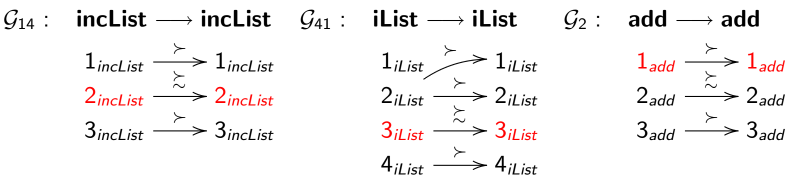

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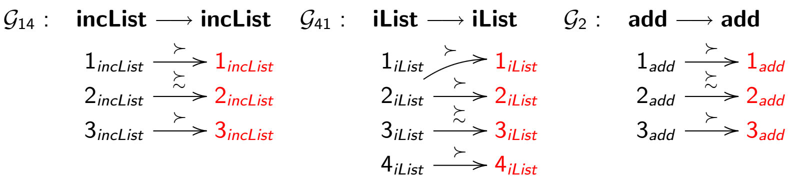

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Concluding remarks

Very fast BTA, scales well to medium-sized Prolog programs

Ensures both local and global termination

Less accurate than previous BTA. . .

Much room for improvement:

- improve accuracy of size-change analysis
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